



# King Saud University Journal of King Saud University – Science

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## ORIGINAL ARTICLE

# Effects on magnetic field in squeezing flow of a Casson fluid between parallel plates



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Received 5 December 2014; accepted 22 March 2015

Available online 27 March 2015

### KEYWORDS

Squeezing flows;  
Variation of parameters  
method (VPM);  
Magneto hydrodynamic  
(MHD);  
Parallel plates;  
Casson fluid

**Abstract** Squeezing flow of an electrically conducting Casson fluid has been taken into account. The laws of conservations under the similarity transformation suggested by Wang (1976) have been used to extract a highly nonlinear ordinary differential equation governing the magneto hydrodynamic (MHD) flow. Resulting equation has been solved analytically by using the variation of parameters method (VPM). A RK-4 numerical solution has also been sought to examine the validity of analytical results. Both the solutions are found to be in an excellent agreement. Convergence of the solution is also discussed. Flow behavior under the modifying involved physical parameters is also discussed and explained in detail with the graphical aid. It is observed that magnetic field can be used as a control phenomenon in many flows as it normalizes the flow behavior. Also, squeeze number plays an important role in these types of problems and an increase in squeeze number increases the velocity profile.

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## 1. Introduction

Many mechanical equipment work under the principle of moving pistons where two plates exhibit the squeezing movement normal to their own surfaces. Electric motors, engines and hydraulic lifters also have this clutching flow in some of their

parts. Due to this practical significance squeezing flow between parallel plates has become one of the most active research fields in fluid mechanics. Its biological applications are also of equal importance. Flow inside syringes and nasogastric tubes is also a kind of squeezing flows.

Foundational work regarding squeezing flows can be named to Stefan (1874) who presented basic formulation of these types of flows under lubrication assumption. After him numbers of scientist have shown their interests toward squeezing flows and have carried out many scientific studies to understand these flows. Some of selected contributions are mentioned in forthcoming lines.

1986 Reynolds (1886) investigated the squeezing flow between elliptic plates. Archibald (1956) considered the same

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problem for rectangular plates. After that several attempts have been made by different researchers to understand squeezing flows in a better way (Grimm, 1976; Wolfe, 1965; Kuzma, 1968; Tichy and Winer, 1970; Jackson, 1962).

Earlier studies on squeezing flows were based on Reynolds equation whose insufficiency for some cases has been shown by Jackson (1962) and Usha and Sridharan (1996). Due to efforts of Birkhoff (1960), Yang (1958) and Wang and Watson (1979) more flexible and useful similarity transforms are now available. These similarity transforms reduce the Navier–Stokes equation into a fourth order nonlinear ordinary differential equation and have further been used in some other investigations as well (Wang, 1976; Laun et al., 1999; Hamdan and Baron, 1992; Nhan, 2000; Rashidi et al., 2008).

Flow of electrically conducting non-Newtonian fluid is a very important phenomenon as in most of the practical situations we have to deal with the flow of conducting fluid which exhibits different behaviors under the influence of magnetic forces. In these cases magneto hydro dynamic (MHD) aspect of the flow is also needed to be considered. Homotopy perturbation solution for Two-dimensional MHD squeezing flow between parallel plates has been determined by Siddiqui et al. (2008). Domairry and Aziz (2009) investigated the same problem for the flow between parallel disks. Recently, Mustafa et al. (2012) examined heat and mass transfer for squeezing flow between parallel plates using the homotopy analysis method (HAM).

In most of realistic models the fluids involved are not simple Newtonian. Complex rheological properties of non-Newtonian fluids cannot be captured by a single model. Different mathematical models have been used to study different types of non-Newtonian fluids. One of such models is known as Casson fluid model. (Mrill et al., 1965; McDonald, 1974) showed that it is the most compatible formulation to simulate blood type fluid flows. It is clear from the literature survey that the squeezing flow of a Casson fluid between the plates moving normal to their own surface is yet to be inspected.

Due to the inherent nonlinearity of the equations governing the fluid flow exact solutions are very rare. Even where they are available immense simplification assumptions have been imposed. Those overly imposed suppositions may not be used for more realistic flows. However to deal with this hurdle many analytical approximation techniques have been developed which are commonly used nowadays (Abbasbandy, 2007a; Abbasbandy, 2007b; Abdou and Soliman, 2005; Noor and Mohyud-Din, 2007; Asadullah et al., 2013; Khan et al., 2012; Ahmed et al., 2014). Variation of parameters method (VPM) is one of these recently developed analytical techniques that have been used to solve different problems (Khan et al., 2014; Noor et al., 2008; Mohyud-Din et al., 2009; Khan et al., 2014a; Khan et al., 2014b; Khan et al., 2014c; Zaidi et al., 2013).

A literature survey reveals that no attempt has ever been made to study the MHD squeezing flow of a Casson fluid. So, in this paper we have presented a comprehensive study for this problem. VPM has been applied to study the nonlinear ordinary differential equation. A numerical solution to the problem has also been sought by using the Runge Kutta order 4 method. Comparison between both the solutions shows that the results obtained by VPM are in excellent agreement with the numerical results.

## 2. Governing equations

We consider an incompressible flow of a Casson fluid between two parallel plates separated by a distance  $z = \pm l(1 - \alpha t)^{1/2} = \pm h(t)$ , where  $l$  is the initial gap between the plates (at a time  $t = 0$ ). Additionally  $\alpha > 0$  corresponds to a squeezing motion of both the plates until they touch each other at  $t = 1/\alpha$ , for  $\alpha < 0$  the plates bear a receding motion and dilate as described in Fig. a. Rheological equation for Casson fluid is defined as under (Nadeem et al., 2012; Nadeem et al., 2013; Nadeem et al., 2014a; Nadeem et al., 2014b; Nadeem et al., 2014c; Ahmed et al., 2013; Casson, 1959; Akber and Khan, 2015; Akbar et al., 2014; Nakamura and Sawada, 1987; Nakamura and Sawada, 1988)

$$\tau_{ij} = \begin{cases} 2\left[\mu_B + \left(\frac{p_y}{2\pi}\right)\right]e_{ij}, & \pi > \pi_c \\ 2\left[\mu_B + \left(\frac{p_y}{2\pi_c}\right)\right]e_{ij}, & \pi_c > \pi \end{cases} \quad (1)$$

where  $\pi = e_{ij}e_{ij}$ , and  $e_{ij}$  is the  $(i, j)$ th component of the deformation rate, i.e.  $\pi$  is the product of the component of deformation rate with itself,  $\pi_c$  is the critical value of the said product,  $\mu_B$  is plastic dynamic viscosity of the non-Newtonian fluid and  $p_y$  is yield stress of slurry fluid.

A constant magnetic field of strength  $M_0$  is imposed perpendicular and relatively fixed to the walls. We are also applying the following assumptions on the flow model:

- The effects of induced magnetic and electric fields produced due to the flow of electrically conducting fluid are negligible.
- No external electric field is present.

Under aforementioned constraints the conservation equations for the flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(1 + \frac{1}{\gamma}\right) \left(2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial x}\right) - \frac{\sigma \beta^2}{\rho} u, \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(1 + \frac{1}{\gamma}\right) \left(2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial y \partial x}\right), \end{aligned} \quad (3)$$

where  $u$  and  $v$  are the velocity components in  $x$  and  $y$ -directions respectively,  $p$  is the pressure,  $\nu = \frac{\mu}{\rho}$  is the dynamic viscosity of the fluid (ratio of kinematic viscosity and density),

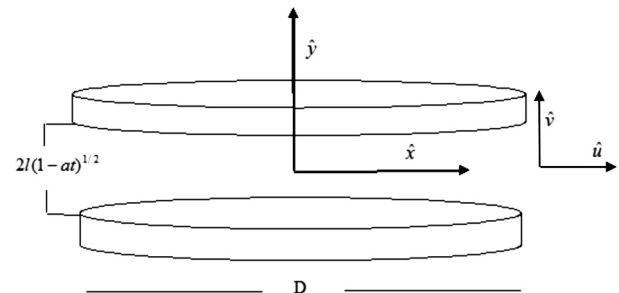


Figure a Schematic diagram for the flow problem.

$\gamma = \mu_B \sqrt{2\pi_c}/p_y$  is the Casson fluid parameter and  $\beta$  is the magnitude of imposed magnetic field.

Supporting conditions for the problem are as follows

$$\begin{aligned} u = 0, \quad v = v_w = \frac{dh}{dt} \quad \text{at } y = h(t), \\ \frac{\partial u}{\partial y} = 0, \quad v = 0 \quad \text{at } y = 0. \end{aligned} \quad (4)$$

We can simplify the above system of equations by eliminating the pressure terms from Eqs. (2) and (3) and using Eq. (1). After cross differentiation and introducing vorticity  $\omega$  we get

$$\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = v \left( 1 + \frac{1}{\gamma} \right) \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) - \frac{\sigma \beta^2}{\rho} \frac{\partial u}{\partial y}, \quad (5)$$

where,

$$\omega = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right). \quad (6)$$

Transform introduced by Wang (1976) for a two-dimensional flow is sated as

$$u = \frac{\alpha x}{[2(1 - \alpha t)]} F'(\eta), \quad (7)$$

$$v = \frac{-\alpha l}{[2(1 - \alpha t)^{1/2}]} F(\eta), \quad (8)$$

where,

$$\eta = \frac{y}{[l(1 - \alpha t)^{1/2}]}. \quad (9)$$

Substituting Eqs. (7)–(9) in Eq. (5) using (6), we obtain a nonlinear ordinary differential equation describing the Casson fluid flow as

$$\begin{aligned} \left( 1 + \frac{1}{\gamma} \right) F^{iv}(\eta) - S(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta)) \\ - F(\eta)F'''(\eta) - M^2 F''(\eta) = 0, \end{aligned} \quad (10)$$

where  $S = \alpha l^2/2v$  denotes the non-dimensional Squeeze number. Using Eqs. (7)–(9) boundary conditions for the problem reduce to

$$F(0) = 0, \quad F''(0) = 0, \quad F(1) = 1, \quad F'(1) = 0. \quad (11)$$

Here, squeeze number  $S$  describes the movement of the plates ( $S > 0$  corresponds to the plates moving apart, while  $S < 0$  corresponds to collapsing movement of the plates). It is pertinent to mention here that for  $M = 0$  and  $\gamma \rightarrow \infty$ , our study reduces to the one obtained by Wang (1976).

Skin friction coefficient is defined as

$$C_f = v \left( 1 + \frac{1}{\gamma} \right) \frac{\left( \frac{\partial u}{\partial y} \right)_{y=h(t)}}{v_w^2}.$$

In terms of Eqs. (7)–(9), we have

$$l^2/x^2(1 - \alpha t) \text{Re}_x C_f = \left( 1 + \frac{1}{\gamma} \right) F'(1). \quad (12)$$

$$\text{Here } \text{Re}_x = \frac{2lv_w^2}{\nu x(1 - \alpha t)^{1/2}}$$

### 3. Solution procedure

Using the standard procedure for VPM (Noor et al., 2008; Mohyud-Din et al., 2009; Khan et al., 2014a; Khan et al., 2014b; Khan et al., 2014c; Zaidi et al., 2013), we can write Eq. (10) as

$$\begin{aligned} F_{n+1}(\eta) = A_1 + A_2\eta + A_3 \frac{\eta^2}{2} + A_4 \frac{\eta^3}{6} - \int_0^\eta \left( \frac{\eta^3}{3!} - \frac{\eta^2 s}{2!} + \frac{\eta s^2}{2!} + \frac{s^3}{3!} \right) \\ \times \left( - \left( \frac{\gamma}{1 + \gamma} \right) \left( S \left( sF(s) + 3F''(s) + F'(s)F''(s) \right) \right) \right) ds, \end{aligned} \quad (13)$$

Here,  $A_1, A_2, A_3$  and  $A_4$  are the constants obtained by taking the highest order linear term of the Eq. (10) and integrating it 4 times to get the final form of the scheme. Using the boundary conditions given in Eq. (11), the above equation can be written as

$$\begin{aligned} F_{n+1}(\eta) = A_2\eta + A_4 \frac{\eta^3}{6} - \int_0^\eta \left( \frac{\eta^3}{3!} - \frac{\eta^2 s}{2!} + \frac{\eta s^2}{2!} + \frac{s^3}{3!} \right) \\ \times \left( - \left( \frac{\gamma}{1 + \gamma} \right) \left( S \left( sF(s) + 3F''(s) + F'(s)F''(s) \right) \right) \right) ds, \end{aligned} \quad (14)$$

with  $n = 0, 1, 2, \dots$ , where  $A_2$  and  $A_4$  are the constants which can be computed by using the boundary conditions  $F(1) = 1$  and  $F'(1) = 0$ , respectively.

First few terms of the solution are given as

$$\begin{aligned} F_0(\eta) &= \left( \frac{\gamma}{1 + \gamma} \right) \left( A_2\eta + A_4 \frac{\eta^3}{6} \right), \\ F_1(\eta) &= \left( \frac{\gamma}{1 + \gamma} \right) \left( A_2\eta + A_4 \frac{\eta^3}{6} \right) - \left( \frac{\gamma}{1 + \gamma} \right) \left( \frac{1}{30} S A_4 + \frac{1}{120} M^2 A_4 \right) \eta^5 \\ &\quad - \left( \frac{\gamma}{1 + \gamma} \right) \left( \frac{1}{2520} S A_4^2 \right) \eta^7, \\ F_2(\eta) &= \left( \frac{\gamma}{1 + \gamma} \right) \left( A_2\eta + A_4 \frac{\eta^3}{6} \right) - \left( \frac{\gamma}{1 + \gamma} \right) \left( \frac{1}{30} S A_4 + \frac{1}{120} M^2 A_4 \right) \eta^5 \\ &\quad - \left( \frac{\gamma}{1 + \gamma} \right) \left( \frac{1}{2520} S A_4^2 + \frac{1}{210} S^2 A_4 - \frac{1}{2520} S M^2 A_4 A_2 \right) \eta^7 \\ &\quad - \left( \frac{\gamma}{1 + \gamma} \right) \left( -\frac{1}{630} S^2 A_2 A_4 + \frac{1}{504} S M^2 A_4 + \frac{1}{5040} M^4 A_4 \right) \eta^9 \\ &\quad - \left( \frac{\gamma}{1 + \gamma} \right) \left( \frac{1}{11340} S^2 A_4^2 - \frac{1}{45360} S^2 A_2 A_4^2 + \frac{1}{60480} M^2 S A_4^2 \right) \eta^9 \\ &\quad - \left( \frac{\gamma}{1 + \gamma} \right) \left( \frac{1}{178200} S^3 A_4^2 - \frac{1}{2494800} S^2 A_4^3 + \frac{1}{356400} S^2 M^2 A_4^2 + \frac{1}{2851200} S M^4 A_4^2 \right) \eta^{11} + \dots \end{aligned} \quad (15)$$

Similarly, the other iterations for the solution can also be computed.

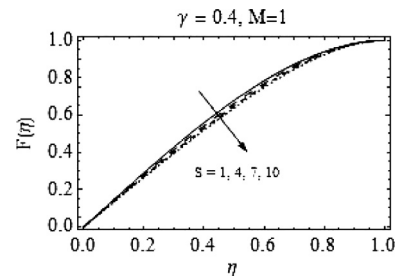


Figure 1 Effects of positive values of  $S$  on  $F(\eta)$ .

#### 4. Results and discussions

In this section the influences of squeeze number  $S$ , Casson fluid parameter  $\gamma$  and the magnetic number  $M$  on the axial ( $F(\eta)$ ) and radial ( $F'(\eta)$ ) velocities are characterized. Figs. 1–16 are displayed for the said purpose. First eight figures are for the case when the plates are moving apart ( $S > 0$ ). Fig. 1 depicts the effects of increasing values of squeeze number  $S$  on the axial velocity  $F(\eta)$ . It is clear, increasing  $S$  results in a decreased axial velocity. Effects of rising  $S$  on radial velocity are shown in Fig. 2. For increasing  $S$ , a decrease in  $F'(\eta)$  is observed when  $\eta \leq 0.5$ . However, there is an increase in  $F'(\eta)$  for  $0.5 < \eta \leq 1$ . Fig. 3 illustrates the behavior of Casson fluid parameter  $\gamma$  on  $F(\eta)$ . Increase in  $\gamma$  decelerates the axial flow. Effects of growing  $\gamma$  on radial velocity are shown in Fig. 4. Increasing  $\gamma$  decreases  $F'(\eta)$  for  $\eta \leq 0.5$  and a rise in  $F'(\eta)$  is observed for  $0.5 < \eta \leq 1$ .

In Figs. 5–8, the consequences of varying magnetic number  $M$  on  $F(\eta)$  and  $F'(\eta)$  are portrayed. It can be observed from Fig. 5 that for increasing  $M$ , there is a decrease in  $F(\eta)$  for slightly lower values of squeeze number  $S$ ; while for  $F'(\eta)$ , the increase in  $M$  provides a velocity profile similar to the case of increasing  $S$ . That is, increasing the magnitude of magnetic field fallouts in a uniform decrease in the velocity. Figs. 7 and 8 are drawn to analyze the effects of magnetic field for slightly higher values of squeeze number  $S$ . The behavior of axial and radial velocities remains almost similar to lower  $S$ .

Figs. 9–16 are presented to study the effects of physical parameters for dilating plates ( $S < 0$ ). In Fig. 9, considerable axial acceleration is observed for falling  $S$ . Fig. 10 demonstrates the effects of decreasing squeeze number on radial velocity. It is clear that  $F'(\eta)$  increases with squeeze rate for  $\eta \leq 0.4$ . A sudden change in  $F'(\eta)$  is observed when  $0.4 < \eta \leq 1$ , i.e. for decreasing values of squeeze number there is a rapid decrease in radial velocity of the fluid. Figs. 11 and 12 show the effects of Casson fluid parameter on axial and

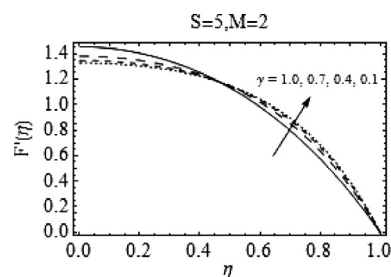


Figure 4 Effects of  $\gamma$  on  $F'(\eta)$ .

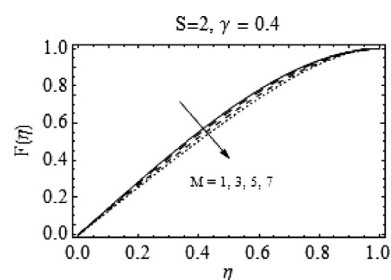


Figure 5 Effects of Mon  $F(\eta)$ .

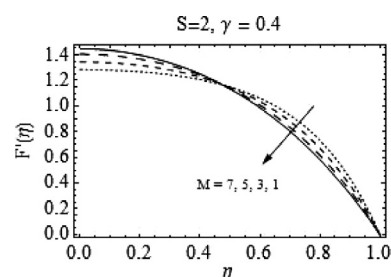


Figure 6 Effects of Mon  $F'(\eta)$ .

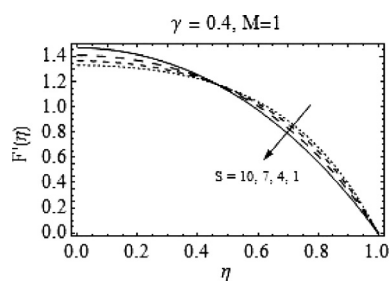


Figure 2 Effects of positive values of  $S$  on  $F'(\eta)$ .

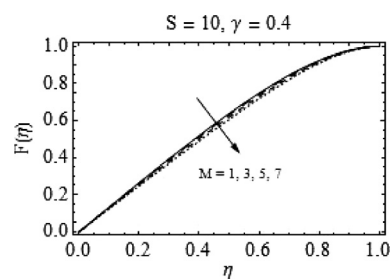


Figure 7 Effects of Mon  $F(\eta)$ .

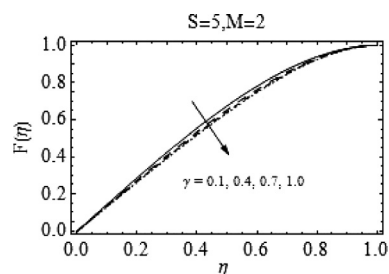


Figure 3 Effects of  $\gamma$  on  $F(\eta)$ .

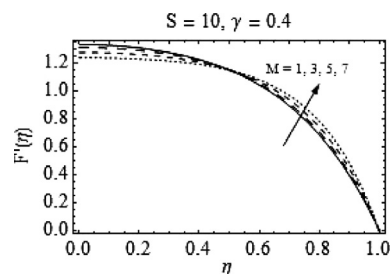
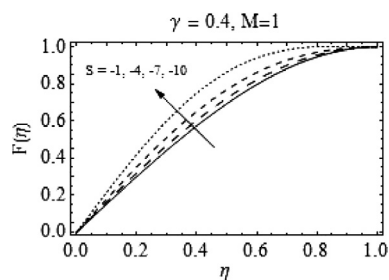
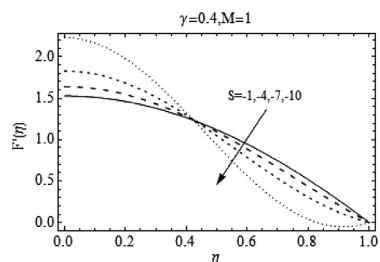
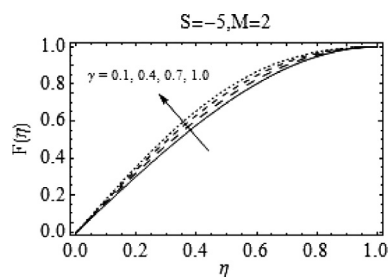
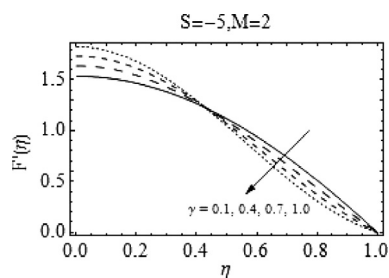
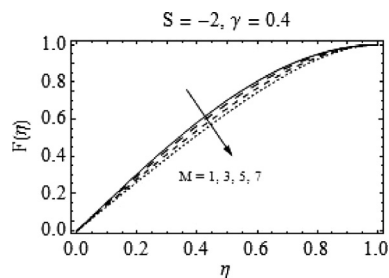
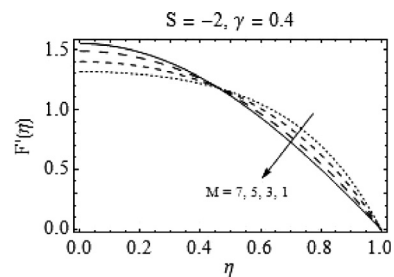
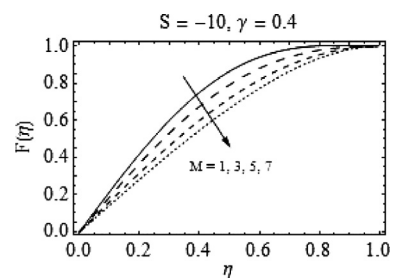
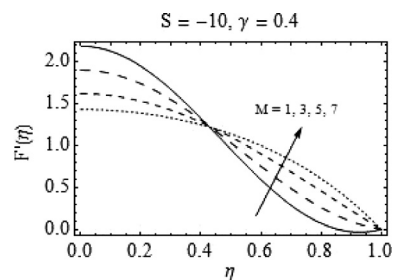


Figure 8 Effects of Mon  $F'(\eta)$ .

Figure 9 Effects of negative values of  $S$  on  $F(\eta)$ .Figure 10 Effects of negative values of  $S$  on  $F'(\eta)$ .Figure 11 Effects of  $\gamma$  on  $F(\eta)$ .Figure 12 Effects of  $\gamma$  on  $F'(\eta)$ .Figure 13 Effects of  $M$  on  $F(\eta)$ .Figure 14 Effects of  $M$  on  $F'(\eta)$ .Figure 15 Effects of  $M$  on  $F(\eta)$ .Figure 16 Effects of  $M$  on  $F'(\eta)$ .**Table 1** Convergence of VPM solution for  $\gamma = 0.4$  and  $M = 1$ .

Order of Approximations	S = 5		S = -5	
	$A_2 = f'$	$A_4 = f'''$	$A_2 = f'$	$A_4 = f'''$
1	1.378126	-1.526101	1.797777	-6.800000
2	1.351366	-1.331071	1.730490	-6.059060
3	1.347469	-1.308010	1.733885	-6.092031
4	1.347157	-1.306332	1.733850	-6.091734
5	1.347136	-1.306259	1.733850	-6.091732
6	1.347136	-1.306257	1.733850	-6.091732
8	1.347136	-1.306257	1.733850	-6.091732
10	1.347136	-1.306257	1.733850	-6.091732
Numerical (RK-4)	1.347136	-1.306257	1.733850	-6.091732

radial velocities respectively. Almost an identical behavior is observed for Casson fluid parameter  $\gamma$  and the squeeze number  $S$  when plates are coming together.

Figs. 13–16 present behavior of the flow when plates are coming together ( $S < 0$ ) and the magnetic number is varying. In Fig. 13, the effects of increasing magnetic number on  $F(\eta)$  are displayed and a decrease in  $F(\eta)$  is observed for slightly



**Table 2** Comparison between the numerical and VPM solutions for  $\gamma = 0.4$  and  $M = 1$ .

$\eta$	$S = 5$				$S = -5$			
	$F(\eta)$ VPM	Numerical	$F'(\eta)$ VPM	Numerical	$F(\eta)$ VPM	Numerical	$F'(\eta)$ VPM	Numerical
0	0	0	1.359393	1.359393	0	0	1.677216	1.677216
0.1	0.139081	0.139081	1.348452	1.348452	0.166839	0.166839	1.650804	1.650804
0.2	0.276358	0.276358	1.357517	1.357517	0.328444	0.328444	1.572994	1.572994
0.3	0.409918	0.409918	1.310148	1.310148	0.479861	0.479861	1.447971	1.447971
0.4	0.537628	0.537628	1.239953	1.239953	0.616685	0.616685	1.282424	1.282424
0.5	0.657014	0.657014	1.142869	1.142869	0.735286	0.735286	1.085120	1.085120
0.6	0.765125	0.765125	1.013414	1.013414	0.832992	0.832992	0.866366	0.866366
0.7	0.858383	0.858383	0.844480	0.844480	0.908218	0.908218	0.637365	0.637365
0.8	0.932408	0.932408	0.627096	0.627096	0.960506	0.960506	0.409532	0.409532
0.9	0.981819	0.981819	0.350136	0.350136	0.990529	0.990529	0.193804	0.193804
1.0	1	0	0	0	1	1	0	0

**Table 3** Comparison of VPM and numerical solutions with existing results for  $M = 0$  and  $\gamma \rightarrow \infty$ .

$S \downarrow$	Present results (HPM)	Present results (RK-4)	Wang (1976)
-0.9780	-2.1915	-2.1915	-2.235
-0.4977	-2.6193	-2.6193	-2.6272
-0.09998	-2.9277	-2.9277	-2.9279
0	-3.000	-3.000	-3.000
0.09403	-3.0663	-3.0663	-3.0665
0.4341	-3.2943	-3.2943	-3.2969
1.1224	-3.708	-3.708	-3.714

**Table 4** Numerical values for skin friction coefficient.

S	$\gamma$	M	$\left(1 + \frac{1}{\gamma}\right) F''(1)$
-5.0	0.4	1.0	-6.298708
-3.0			-8.320727
-1.0			-9.970376
1.0			-11.376240
3.0			-12.610669
5.0			-13.718095
-3.0	0.1		-30.991005
	0.3		-10.873387
	0.5		-6.771549
3.0	0.1		-35.260196
	0.3		-15.149577
	0.5		-11.078736
-3.0	0.4	2	-13.101572
		4	-14.908219
		6	-17.501183
3.0	0.4	2	-9.038196
		4	-11.531983
		6	-14.819321

higher values of  $S$ . Fig. 14 gives us a graphical demonstration of  $F(\eta)$  for growing magnetic number. It shows  $F(\eta)$  decreases for  $\eta \leq 0.4$  however for  $0.4 < \eta \leq 1$  it behaves otherwise, i.e. for increasing values of magnetic number, there is a rapid increase in radial velocity of the fluid. A similar type of behavior is observed for increasing magnetic number when  $S = -10$  with more prominent effects. Similarly, in Fig. 16, a quite rapid change can be observed for increasing values of magnetic

number. Also, with a decrease in squeeze number the backflow may emerge and a strong magnetic field is required to enhance the flow as shown in Fig. 16.

It is important to check the convergence of the series solution obtained in Eq. (15). For this purpose, the numerical values of unknown constants  $A_2$  and  $A_4$  are computed in Tables 1. It is pertinent to mention that VPM converges only after 6 approximations. Obtained analytical results are also compared with the ones acquired by using the RK-4 method and an excellent agreement is found.

A comparison between the analytical and numerical solutions is shown in Table 2 for the axial and radial velocities. It can clearly be seen that the solutions agree very well. Table 3 compares the present results with some already existing solutions in the literature and again an excellent agreement is observed.

Table 4 presents the numerical values of skin friction coefficient. It can be seen that for all increasing parameters an increase in the magnitude of skin friction coefficient is obtained.

## 5. Conclusions

MHD squeezing flow of a non-Newtonian namely Casson fluid is presented between parallel plates. Governing equations are reduced to a single ordinary differential equation using a similarity transform. Two cases are considered, i.e. one when plates are moving apart and other when plates are coming closer. VPM is used to solve the equations governing the flow. Effects of emerging parameters on flow are demonstrated graphically and a comprehensive discussion is provided. A numerical solution is also obtained using RK-4 method to compare the results obtained by VPM and an excellent agreement is found among the solutions. It can be concluded from the graphs that a strong magnetic field can be used to enhance the flow when plates are coming together and squeeze number increases the velocity profile for both the cases, i.e., when plates are coming closer and when plates are going apart.

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